

Energy associated with charged dilaton black holes

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Abstract

It is known that certain properties of charged dilaton black holes depend on a free parameter β which controls the strength of the coupling of the dilaton to the Maxwell field. We obtain the energy associated with static spherically symmetric charged dilaton black holes for arbitrary value of the coupling parameter and find that the energy distribution depends on the value of β . With increasing radial distance, the energy in a sphere increases for $\beta = 0$ as well as for $\beta < 1$, decreases for $\beta > 1$, and remains constant for $\beta = 1$. However, the total energy turns out to be the same for all values of β .

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Since the last few years there has been a growing interest in obtaining dilaton black hole solutions and studying their properties which has led to several new insights about black holes ([1]-[11]). Garfinkle, Horowitz, and Strominger(GHS)[2] obtained a nice form of static spherically symmetric charged dilaton black hole solutions which exhibit several different properties compared to the Reissner-Nordström (RN) black holes. The dilaton has an important role in these solutions and many of their interesting features have their origin in the coupling between the Maxwell and the dilaton fields. GHS considered the action ([2]-[3])

$$S = \int d^4x \sqrt{-g} \left[-R + 2(\nabla\Phi)^2 + e^{-2\beta\Phi} F^2 \right], \quad (1)$$

where R is the Ricci scalar, Φ is the dilaton field, F_{ab} is the electromagnetic field tensor, and β is a dimensionless free parameter which regulates the coupling between the dilaton and the Maxwell fields. It is clear that a change in the sign of the free parameter β is the same as a change in the sign of the dilaton field Φ ; therefore it is adequate to discuss only nonnegative values of β . (1) gives the action for the Einstein-Maxwell-scalar theory for $\beta = 0$ and the action for the Kaluza-Klein theory for $\beta = \sqrt{3}$. Moreover, $\beta = 1$ in (1) gives the action which is a part of the low-energy action of string theory. Varying (1) gives the equations of motion:[3]

$$\begin{aligned} \nabla_i (e^{-2\beta\Phi} F^{ik}) &= 0, \\ \nabla^2\Phi + \frac{\beta}{2} e^{-2\beta\Phi} F^2 &= 0, \\ R_{ik} &= 2\nabla_i\Phi\nabla_k\Phi + 2e^{-2\beta\Phi} F_{ia}F_k{}^a - \frac{1}{2}g_{ik}e^{-2\beta\Phi} F^2. \end{aligned} \quad (2)$$

GHS obtained static spherically symmetric charged dilaton black hole solutions of these equations given by the line element,

$$ds^2 = Bdt^2 - A dr^2 - Dr^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (3)$$

the dilaton field Φ

$$e^{2\Phi} = \left[1 - \frac{r_-}{r} \right]^{(1-\sigma)/\beta}, \quad (4)$$

and the electromagnetic field tensor component

$$F_{tr} = \frac{Q}{r^2}, \quad (5)$$

where

$$B = A^{-1} = \left(1 - \frac{r_+}{r} \right) \left(1 - \frac{r_-}{r} \right)^\sigma, \quad (6)$$

$$D = \left(1 - \frac{r_-}{r}\right)^{1-\sigma}, \quad (7)$$

and

$$\sigma = \frac{1 - \beta^2}{1 + \beta^2}. \quad (8)$$

r_+ and r_- are related to the mass and charge parameters, M and Q , through

$$\begin{aligned} 2M &= r_+ + \sigma r_-, \\ Q^2 (1 + \beta^2) &= r_+ r_-. \end{aligned} \quad (9)$$

For $\beta = 0$ the solution yields the well known RN solution of the Einstein-Maxwell equations. The surface $r = r_+$ is an event horizon for all values of β .

There are several properties of charged dilaton black holes which depend crucially on the free parameter β . The maximum charge which can be carried by a charged dilaton black hole (for a given mass) depends on the value of β (see [10]). However, for a given mass there exists an extremal limit for all values of β . When $\beta \neq 0$, the surface $r = r_-$ is a curvature singularity and when $\beta = 0$ (the case of the RN solution) the surface $r = r_-$ is a nonsingular inner horizon[3]. The entropy and temperature of these black holes depend on β (see [10]). For the extremal black holes ($r_+ = r_-$) the entropy is finite for $\beta = 0$ and is zero for $\beta \neq 0$, and the temperature is zero for $\beta < 1$, is finite (the same as for a Schwarzschild black hole) for $\beta = 1$, and is infinite for $\beta > 1$. Holzhey and Wilczek[10] showed that, for $\beta > 1$, infinite potential barriers form around extremal charged dilaton black holes leading to the interpretation that these extremal black holes behave like elementary particles. They discussed that the extremal charged dilaton black holes appear to be extended spherical objects for $\beta < 1$ and remarked that one can hardly refrain from observing that a string is the intermediate case between a spherical membrane and a point. Horne and Horowitz[3] and Shiraishi[4] obtained charged rotating dilaton black hole solutions for small values of the rotation parameter. They calculated the gyromagnetic ratio for these black holes and found that it depends on the free parameter β .

The energy associated with the Schwarzschild and the RN black holes is well discussed in the literature (see [12]-[13]) and references therein). The energy of the Schwarzschild black hole is confined to its interior whereas the energy of the RN black hole is shared by its interior as well as exterior. One of the present authors and Parikh[14] investigated the energy of a static spherically symmetric charged dilaton black hole for $\beta = 1$ and found the interesting result that, similar to the case of the Schwarzschild black hole and unlike the RN black hole, the entire energy is confined to its interior with no energy shared by the exterior of the black hole. As several features of the charged dilaton black holes depend crucially on the coupling parameter β , it is of interest to study the energy associated with charged dilaton black holes for arbitrary values of β to see (a) what the energy distribution is for

$\beta < 1$ as well as for $\beta > 1$, and (b) whether or not the energy is confined to the black hole interior for any other value of β (apart from $\beta = 1$).

Since the outset of the general theory of relativity several prescriptions for calculating the energy and momentum have been proposed, e.g. the energy-momentum complex of Einstein[12]

$$\Theta_i{}^k = \frac{1}{16\pi} H_{i, l}{}^{kl}, \quad (10)$$

where

$$H_i{}^{kl} = \frac{g_{in}}{\sqrt{-g}} \left[-g \left(g^{kn} g^{lm} - g^{ln} g^{km} \right) \right]_{,m}. \quad (11)$$

$H_i{}^{kl}$ is antisymmetric in its contravariant indices. Latin indices take values from 0 to 3 (x^0 is the time coordinate). $\Theta_i{}^k$, given by (10), satisfies the local conservation laws

$$\frac{\partial \Theta_i{}^k}{\partial x^k} = 0, \quad (12)$$

where

$$\Theta_i{}^k = \sqrt{-g} \left(T_i{}^k + \vartheta_i{}^k \right). \quad (13)$$

$T_i{}^k$ is the symmetric energy-momentum tensor due to matter and all nongravitational fields and $\vartheta_i{}^k$ is the energy-momentum pseudotensor due to the gravitational field only. The energy and momentum components are given by

$$P_i = \frac{1}{16\pi} \int \int \int H_i{}^{0\alpha}{}_{,\alpha} dx^1 dx^2 dx^3, \quad (14)$$

where the Greek index α takes values from 1 to 3. P_0 stands for the energy (say E), and P_1, P_2, P_3 are the momentum components.

The Einstein prescription for obtaining the energy and momentum associated with asymptotically flat spacetimes gives meaningful result if calculations are carried out in those coordinates in which the metric g_{ik} approaches the Minkowski metric η_{ik} at great distance from the system under investigation. These coordinates are usually called quasi-Cartesian or quasi-Minkowskian. Transforming the line element (3) to these coordinates according to

$$\begin{aligned} x &= r \sin\theta \cos\varphi, \\ y &= r \sin\theta \sin\varphi, \\ z &= r \cos\theta, \end{aligned} \quad (15)$$

one gets

$$ds^2 = Bdt^2 - D(dx^2 + dy^2 + dz^2) - \frac{A-D}{r^2}(xdx + ydy + zdz)^2. \quad (16)$$

The determinant $g \equiv |g_{ik}|$ is

$$g = - \left(\frac{r}{r-r_-} \right)^{2(\sigma-1)}, \quad (17)$$

and the nonvanishing contravariant components of the metric tensor are

$$\begin{aligned} g^{00} &= \frac{r^{\sigma+1}}{(r-r_-)^\sigma(r-r_+)}, \\ g^{11} &= \frac{(r-r_-)^{\sigma-1}}{r^{\sigma+3}} \left[x^2 (rr_- + rr_+ - r_-r_+) - r^4 \right], \\ g^{22} &= \frac{(r-r_-)^{\sigma-1}}{r^{\sigma+3}} \left[y^2 (rr_- + rr_+ - r_-r_+) - r^4 \right], \\ g^{33} &= \frac{(r-r_-)^{\sigma-1}}{r^{\sigma+3}} \left[z^2 (rr_- + rr_+ - r_-r_+) - r^4 \right], \\ g^{12} &= \frac{xy(r-r_-)^{\sigma-1}}{r^{\sigma+3}} [r(r_- + r_+) - r_-r_+], \\ g^{23} &= \frac{yz(r-r_-)^{\sigma-1}}{r^{\sigma+3}} [r(r_- + r_+) - r_-r_+], \\ g^{31} &= \frac{zx(r-r_-)^{\sigma-1}}{r^{\sigma+3}} [r(r_- + r_+) - r_-r_+]. \end{aligned} \quad (18)$$

We are interested in calculating the energy and therefore the required components of H_i^{kl} are

$$\begin{aligned} H_0^{01} &= \frac{2x}{r^4} [r(\sigma r_- + r_+) - \sigma r_-r_+], \\ H_0^{02} &= \frac{2y}{r^4} [r(\sigma r_- + r_+) - \sigma r_-r_+], \\ H_0^{03} &= \frac{2z}{r^4} [r(\sigma r_- + r_+) - \sigma r_-r_+]. \end{aligned} \quad (19)$$

By using (19) with (9) in (14), applying the Gauss theorem, and then evaluating the integral over the surface of a sphere of radius r , we get

$$E(r) = M - \frac{Q^2}{2r} (1 - \beta^2). \quad (20)$$

Thus we find that, like several other features of the charged dilaton black holes, the energy distribution depends on the value of the coupling parameter β . $\beta = 0$ in (20) gives the

energy distribution in the RN field[13]. In the present investigation we find that *only for* $\beta = 1$ is the energy confined to its interior, and that for all other values of β the energy is shared by the interior and exterior of the black holes. The total energy of the charged dilaton black holes is independent of β and is given by the mass parameter of the black hole. With increasing radial distance, $E(r)$ increases for $\beta = 0$ (RN metric) as well as for $\beta < 1$, decreases for $\beta > 1$, and remains constant for $\beta = 1$.

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